Strong Recursive Feasibility in Model Predictive Control of Biped Walking

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Continuous Adaptation to a dynamic environment

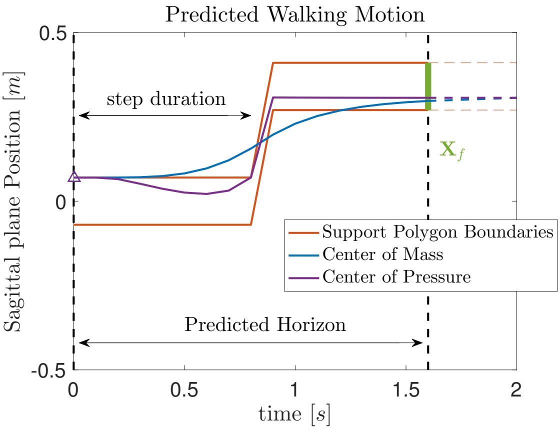
with Model Predictive Control (MPC)

- Constraints satisfaction: **feasibility**, is classically guaranteed **recursively** [Mayne, 2014]
- MPC scheme adapts the robot to a dynamic environment (e.g. collision avoidance)
- We aim therefore at guaranteeing **strong recursive feasibility** [Kerrigan, 2001]

A capturable terminal constraint

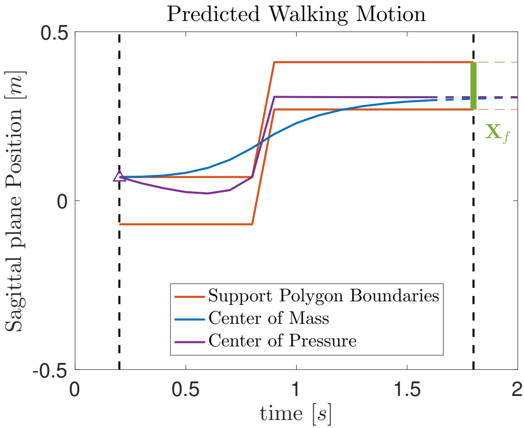
- A **terminal constraint** [Mayne, 2014] guarantees recursive feasibility: the system remains feasible indefinitely after the end of the horizon
- Passive Safety: stop before any collision occurs [Bouraine et al., 2011]
- A **capturable terminal constraint** [Bohorquez et al., 2016]: the robot stops after a given number of steps

When the robot is not planning to stop



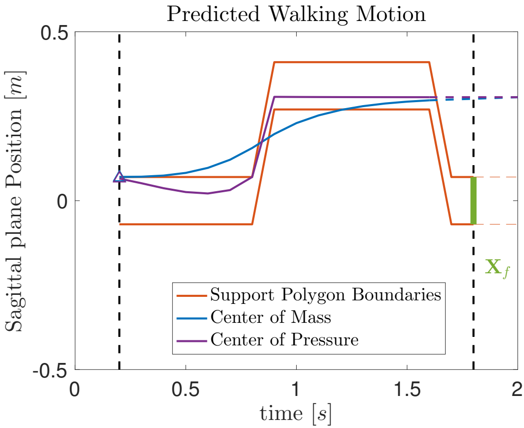
A capturability terminal constraint (that defines a terminal constraint set $\mathbb{X}_f$) makes sure that the motion of the legged robot remains feasible indefinitely.

When the robot is not planning to stop



When the prediction horizon advances,
thanks to the terminal constraint set $\mathbb{X}_f$,
we **know for sure** the MPC scheme remains feasible.

When the robot is not planning to stop



When the robot considers making a new step,
with such a sudden change,
we **don't know for sure** the MPC scheme remains feasible.

Strong Recursive Feasibility in MPC

linear time-invariant discrete-time system:

$$x_{i+1} = Ax_i + Bu_i \ , \tag{1}$$

a time-varying set of constraints:

$$E_i x_i + F_i \mathcal{U}_i \leq d_i \, , \mathcal{U}_i = \{u_{(i|i)}, u_{(i+1|i)}, \ldots, u_{(i+N|i)}\}. \tag{2}$$

set of solutions:

$$\mathcal{W}_i \triangleq \{(x_i, \mathcal{U}_i) \mid E_i x_i + F_i \mathcal{U}_i \leq d_i\} \ . \tag{3}$$

By definition, a **convex polytope**. In our case, **closed**.

Strong Recursive Feasibility in MPC

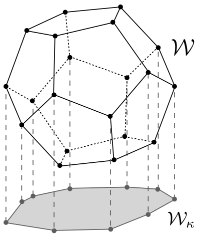
Two useful projections of $\mathcal{W}_i$ are:

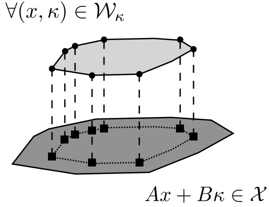
$$(\mathcal{W}_\kappa)_i \triangleq \{(x_i, \kappa) \,|\, \exists \mathcal{U}_i \text{ s.t. } (x_i, \mathcal{U}_i) \in \mathcal{W}_i \wedge u_{(i|i)} = \kappa\}$$
$$\text{and } \mathcal{X}_i \triangleq \{x_i \,|\, \exists \mathcal{U}_i \text{ s.t. } (x_i, \mathcal{U}_i) \in \mathcal{W}_i\} \ . \tag{4}$$

Definition (Strong Recursive Feasibility [Kerrigan, 2001])

The MPC scheme is strongly recursive feasible if and only if $\forall i$

$$\forall\, (x_i, \kappa) \in (\mathcal{W}_\kappa)_i \ , \ Ax_i + B\kappa \in \mathcal{X}_{i+1} \ . \tag{5}$$





Numerical evidence of Strong Recursive Feasibility

- Solve Linear Program (LP):

$$\begin{aligned} \min_{x_i,\mathcal{U}_i} \quad & \gamma^T \begin{bmatrix} x_i \\ \mathcal{U}_i \end{bmatrix} \\ \text{s.t.} \quad & \begin{bmatrix} E_i & F_i \end{bmatrix} \begin{bmatrix} x_i \\ \mathcal{U}_i \end{bmatrix} \leq d_i \ . \end{aligned} \tag{6}$$

  Randomly chosen directions γ provide a random selection of **vertices** of $\mathcal{W}_i$
- Each vertex projected in $(\mathcal{W}_\kappa)_i$
- Check (5) verifying the following LP

$$\begin{aligned} \min_{\mathcal{U}_{i+1}} \quad & \psi^T \mathcal{U}_{i+1} \\ \text{s.t.} \quad & E_{i+1}\left(Ax_i + B\kappa\right) + F_{i+1}\mathcal{U}_{i+1} \leq d_{i+1} \end{aligned} \tag{7}$$

  has a solution

Dynamics of Walking

Automatic Foot Step Placement algorithm for Walking Motion [Herdt et al., 2010]

motion of the Center of Mass (CoM) c as a triple integrator:

$$\hat{c}_{i+1} = A\hat{c}_i + B\dddot{c}_i \ , \tag{8}$$

time-varying set of constraints:

- CoM is linearly related to the Center of Pressure (CoP).
- CoP within the support polygon.
- CoM position constrained by the maximal leg length.
- Not crossing legs while walking.
- Capturable constraint.

They define **a closed convex polytope**.

Evaluation of randomly selected vertices

Before a new step appears at the end of the prediction horizon (time t)

- `linprog` in MATLAB to solve LPs: 6 million random selection of vertices (duplicates eliminated)
- Kinematic Parameters based on HRP-2
- Sampling time $T = 0.1\ [s]$
- Step duration $0.8\ [s]$

Prediction Horizon N	Vertices (x_t, κ) $\exists \mathcal{U}_{t+1}$	$\nexists \mathcal{U}_{t+1}$
8	183	182
9	278	578
10	434	247
11	377	287
12	268	162
13	183	59
14	225	0
15	204	0
16	228	0
17	832	0
18	614	0
19	450	0
20	339	0
21	342	0
22	217	0
23	488	0
24	433	0

Conclusions and Future Directions

- Despite the sudden change of the terminal region set (when a new step is added in the prediction horizon), there is a numerical evidence that the MPC for legged robot is strongly recursive feasible from $N = 14$.

Future Directions:

- Why Capturability constraint? Robot can stop by simply stepping in place.

References

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