Strong Recursive Feasibility in Model Predictive Control of Biped Walking

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Continuous Adaptation to a dynamic environment with Model Predictive Control (MPC)

- Constraints satisfaction: **feasibility**, is classically guaranteed recursively [Mayne, 2014]
- MPC scheme adapts the robot to a dynamic environment (e.g. collision avoidance)
- We aim therefore at guaranteeing **strong recursive feasibility** [Kerrigan, 2001]
A capturable terminal constraint

- A **terminal constraint** [Mayne, 2014] guarantees recursive feasibility: the system remains feasible indefinitely after the end of the horizon
- Passive Safety: stop before any collision occurs [Bouraine et al., 2011]
- A **capturable terminal constraint** [Bohorquez et al., 2016]: the robot stops after a given number of steps
When the robot is not planning to stop

A capturability terminal constraint (that defines a terminal constraint set $X_f$) makes sure that the motion of the legged robot remains feasible indefinitely.
When the robot is not planning to stop

When the prediction horizon advances, thanks to the terminal constraint set $X_f$, we know for sure the MPC scheme remains feasible.
When the robot is not planning to stop

When the robot considers making a new step, with such a sudden change, we don’t know for sure the MPC scheme remains feasible.
Strong Recursive Feasibility in MPC

linear time-invariant discrete-time system:

\[ x_{i+1} = Ax_i + Bu_i \]  \hspace{1cm} (1)

a time-varying set of constraints:

\[ E_i x_i + F_i U_i \leq d_i , \quad U_i = \{ u(i|i), u(i+1|i), \ldots, u(i+N|i) \} \]  \hspace{1cm} (2)

set of solutions:

\[ \mathcal{W}_i \triangleq \{ (x_i, U_i) \mid E_i x_i + F_i U_i \leq d_i \} \]  \hspace{1cm} (3)

By definition, a convex polytope. In our case, closed.
Strong Recursive Feasibility in MPC

Two useful projections of $\mathcal{W}_i$ are:

\[
(\mathcal{W}_k)_i \triangleq \{ (x_i, \kappa) \mid \exists U_i \text{ s.t. } (x_i, U_i) \in \mathcal{W}_i \land u(i|i) = \kappa \}
\]

and $\mathcal{X}_i \triangleq \{ x_i \mid \exists U_i \text{ s.t. } (x_i, U_i) \in \mathcal{W}_i \}$.

Definition (Strong Recursive Feasibility [Kerrigan, 2001])

The MPC scheme is strongly recursive feasible if and only if $\forall i$

\[
\forall (x_i, \kappa) \in (\mathcal{W}_k)_i, \ Ax_i + B\kappa \in \mathcal{X}_{i+1}.
\]

\[
\forall(x, \kappa) \in \mathcal{W}_k
\]

\[
Ax + B\kappa \in \mathcal{X}
\]
Numerical evidence of Strong Recursive Feasibility

- Solve Linear Program (LP):

\[
\begin{align*}
\min_{x_i, U_i} & \quad \gamma^T \begin{bmatrix} x_i \\ U_i \end{bmatrix} \\
\text{s.t.} & \quad [E_i \quad F_i] \begin{bmatrix} x_i \\ U_i \end{bmatrix} \leq d_i.
\end{align*}
\] (6)

Randomly chosen directions \( \gamma \) provide a random selection of vertices of \( \mathcal{W}_i \)

- Each vertex projected in \( (\mathcal{W}_\kappa)_i \)

- Check (5) verifying the following LP

\[
\begin{align*}
\min_{U_{i+1}} & \quad \psi^T U_{i+1} \\
\text{s.t.} & \quad E_{i+1} (Ax_i + B\kappa) + F_{i+1} U_{i+1} \leq d_{i+1}
\end{align*}
\] (7)

has a solution
Dynamics of Walking

Automatic Foot Step Placement algorithm for Walking Motion [Herdt et al., 2010]

motion of the Center of Mass (CoM) $c$ as a triple integrator:

$$\hat{c}_{i+1} = A\hat{c}_i + B\ddot{c}_i ,$$

(8)

time-varying set of constraints:

- CoM is linearly related to the Center of Pressure (CoP).
- CoP within the support polygon.
- CoM position constrained by the maximal leg length.
- Not crossing legs while walking.
- Capturable constraint.

They define a closed convex polytope.
Evaluation of randomly selected vertices

Before a new step appears at the end of the prediction horizon (time $t$)

- `linprog` in MATLAB to solve LPs: 6 million random selection of vertices (duplicates eliminated)
- Kinematic Parameters based on HRP-2
- Sampling time $T = 0.1 \text{ [s]}$
- Step duration $0.8 \text{ [s]}$

<table>
<thead>
<tr>
<th>Prediction Horizon $N$</th>
<th>Vertices $(x_t, \kappa)$ $\exists U_{t+1}$</th>
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Conclusions and Future Directions

- Despite the sudden change of the terminal region set (when a new step is added in the prediction horizon), there is a numerical evidence that the MPC for legged robot is strongly recursive feasible from $N = 14$.

Future Directions:

- Why Capturability constraint? Robot can stop by simply stepping in place.
References


N. Bohorquez, A. Sherikov, D. Dimitrov, and P.-B. Wieber, Safe navigation strategies for a biped robot walking in a crowd, in 2016 IEEE-RAS 16th International Conference on Humanoid Robots (Humanoids)
